Control of Vehicle Dynamics Considering Guideway Design Criteria

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The ride quality of a transportation vehicle is dependent on the effectiveness of the suspension system to filter out vibration due to guideway roughness and other sources such as wind gusts. To meet ride comfort specifications without unreasonable stroke requirements, it is sometimes necessary to compensate the suspension design with some form of either active or passive control. This paper addresses the subject of relating the tradeoff in ride comfort for suspension stroke to specific guideway profile design tolerances and constraints. Specifically, approximate lower bounds of RMS acceleration for any vehicle suspension system as a function of RMS suspension stroke are computed for specific guideway profile elevation variation constraints and construction surveying accuracies. To demonstrate the sensitivity of control requirements to guideway design parameters, example calculations are performed with a model which is representative of a broad class of air cushion suspensions.

Nomenclature

c = statistical roughness parameter
f = frequency, Hz
$G_a(f)$ = vehicle suspension transfer function relating vertical acceleration to guideway irregularities
$G_r(f)$ = vehicle suspension transfer function, relating suspension stroke to guideway irregularities
h = distance between terrain elevation check points, ft
h_b = distance between surveying check points for actually laying out and constructing the design profile, ft
$\overline{h}$ = interval ratio h_b/h
j = $(-1)^{1/2}$
s = LaPlace transform variable
$S_a(f)$ = passenger acceleration power spectral density, ft^2/cycle sec^3
$S_r(f)$ = suspension stroke power spectral density, ft^2 sec/cycle
$S_v(f)$ = temporal guideway power spectral density, ft^2 sec/cycle
$S_w(f)$ = power spectral density of the normalized control flow, sec/cycle
$S_y(\Omega)$ = spatial power spectral density of the profile elevation Y_0, ft^3/cycle
V = vehicle velocity, ft/sec
ΔW_c = incremental control mass flow rate
W_e = equilibrium mass flow rate to air cushions
ΔY = vertical displacement of vehicle, ft
$\Delta \ddot{Y}$ = vertical acceleration of vehicle, ft/sec^2
ΔY_r = suspension vertical stroke or travel, ft
ΔY_0 = perturbations in guideway amplitude, ft
λ = irregularity wavelength, ft/cycle
σ = standard deviation of the random terrain elevations
σ_b = standard deviation of the random surveying errors
$\overline{\sigma}$ = standard deviation ratio, σ_b/σ
ω_n = undamped natural frequency of vehicle suspension, Hz
Ω = spatial frequency, rad/ft

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I. Introduction

THE ride quality of a transportation vehicle is dependent on the effectiveness of the suspension system to filter out vibration due to guideway roughness and other sources such as wind gusts. This paper specifically addresses the interaction between a vehicle with near optimum one-dimensional heave dynamics and a guideway with irregularities consisting of natural terrain profile variations and surveying (construction) inaccuracies. Ride comfort is dependent on the RMS force level transmitted to the vehicle passenger compartment. As the suspension natural frequency is decreased, this RMS force level is decreased and the required stroke of the unsprung mass is increased. Therefore, some form of active or passive suspension control may be required to compensate the suspension design in order to meet ride comfort specifications without unreasonable stroke requirements.

This paper examines the tradeoff relating ride comfort specifications, suspension stroke and specific guideway profile design criteria. The results are based on the suspension system model derived in Ref. 1. Since the guideway characteristics used in this paper approach those of Ref. 1 in the limit, the results of this study are near optimal and, hence, are representative of upper bounds in performance that can be achieved with any vehicle suspension system. The roughness characteristics for the guideway are defined by guideway con-

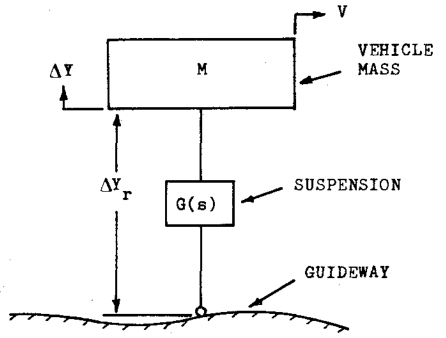

Fig. 1 Schematic of a one-dimensional vehicle suspension system.

struction tolerances and constraints in terms of power spectral densities developed in Ref. 2.

A vehicle suspension schematic is shown in Fig. 1. The primary suspension supports the vehicle above the guideway and prevents excessive vehicle guideway contact. The stiffness of the secondary suspension is relatively soft compared to that of the primary suspension since it serves to isolate the passenger compartment from external disturbances.

The transfer function relating the perturbations in displacement of the unsprung mass ΔY and the perturbations in guideway amplitude ΔY_0 is derived in Ref. 1. This transfer function represents the Wiener filter that minimizes the performance index given by

$$I = \langle \Delta \ddot{Y} \rangle^2 + \omega_n^4 \langle \Delta Y_r \rangle^2$$

Thus it yields a minimum mean square suspension stroke $\langle \Delta Y_r \rangle^2$ for a specified maximum allowable mean square acceleration $\langle \Delta \ddot{Y} \rangle^2$.

In regard to vehicles with air cushion and electromagnetic suspensions, this performance index offers many advantages. for example, minimizing $\langle \Delta Y_r \rangle^2$ also serves to minimize the steady state power requirements since minimum equilibrium clearances between the vehicle and the guideway can be achieved as a result of minimum stroke. Also, by minimizing $\langle \Delta \ddot{Y} \rangle^2$, minimum control power is achieved with suspensions utilizing feedback variables that are proportional to the acceleration.

The derivation of the optimum transfer function in Ref. 1 considers one-dimensional heave motion due to guideway irregularities only. Also, it was assumed that the power spectral density (PSD) for the guideway irregularities to be of the form-CV/s^2. Hence the results presented in this paper are only near optimal since the guideway roughness spectra used in the analysis are based on terrain roughness and construction inaccuracies of which the PSD's are of the form $-CV/s^2$ only in certain limiting cases.

The analytical development of the guideway roughness PSD developed in Ref. 2 relates to two specific sources of guideway irregularities. The first source pertains to random terrain characteristics and necessary constraints on the relative elevation variation of adjacent checkpoints or bench marks along a design profile. The model for these irregularities establishes guidelines for determining the degree of terrain leveling or filling needed to achieve a theoretical design profile which would permit the achievement of ride comfort specifications. The roughness spectrum for this terrain model is asymptotic at relatively long wavelengths to C/Ω^2 where the constant C is a function of the constraint on the elevation variation of the guideway profile.

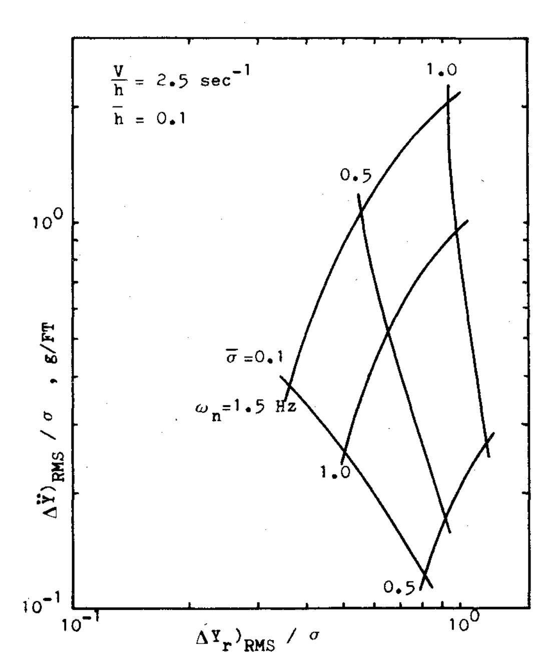

Fig. 2 Effects of guideway design tolerances and constraints on ride comfort and suspension stroke.

The second irregularity source results from errors associated with measurement and/or surveying inaccuracies in actually constructing the desired guideway profile. For example, irregularities resulting from random settlement of elevated guideways are typical construction or maintenance errors. Since these two sources of irregularities are independent, the two models can be added to obtain the total roughness spectrum. Also, since the suspension model just described in near optimum, this approach gives an approximate upper boundary for allowable construction tolerances and constraints. It is recognized that other irregularity sources such as guideway flexibility may significantly influence the roughness spectrum.[3-5] These additional sources are not considered in this analysis however.

Using the suspension model and the guideway roughness spectral models already described, values for RMS vehicle acceleration and RMS suspension stroke are formulated as functions of guideway construction tolerances and constraint magnitudes. This allows the determination of the most effective manipulation of guideway design and construction tolerances and suspension control requirements in order to obtain an acceptable ride comfort level while maintaining a reasonable suspension stroke requirement.

An example of actual control requirements for an air cushion suspension is presented which demonstrates the significance of the guideway construction tolerances and constraints.

II. Vibration Analysis of Vehicle

In order to determine the effects of guideway design and construction tolerances on the tradeoff existing between passenger ride comfort and suspension system design, vehicle acceleration and suspension stroke are derived as functions of guideway design parameters.

The acceleration spectral density is defined by

$$S_a(f) = |G_a(f)|^2 S_v(f) \tag{1}$$

where f is frequency in hertz; $G_a(f)$ is the transfer function that relates the passengers' acceleration to the guideway disturbances; and $S_y(f)$ is the temporal guideway spectral density which is related to the spatial spectral density $S_y(\Omega)$ as follows:

$$S_v(f) = \frac{1}{V} S_y(\Omega) = \frac{1}{V} S_y\left(\frac{2\pi f}{V}\right) \tag{2}$$

V is the forward velocity of the vehicle.

Assuming that the primary suspension is very stiff dynamically, the secondary suspension transfer function-$G_a(f)$, as derived in Ref. 1, is

$$G_a(f) = [s^2\omega_n^2 / (s^2 + \sqrt{2}\omega_n s + \omega_n^2)]_{s=j2\pi f} \tag{3}$$

where ω_n is the damped natural frequency. Substituting Eq. (3) into Eq. (1) gives

$$S_a(f) = \frac{\omega_n^4}{1 + (\omega_n/2\pi f)^4} S_v(f) \tag{4}$$

Using the results obtained in Ref. 2, the spatial spectral density for the terrain profile model and the surveying inaccuracies is

$$S_y(\Omega) = \frac{4\sigma_b{}^2}{\Omega^4 h_b{}^3}(s - 4\cos\Omega h_b + \cos 2\Omega h_b) + \frac{4\sigma^2}{h^3\Omega^4}(1 - \cos\Omega h) \tag{5}$$

where h represents the distance between elevation check points along the design guideway profile; σ is the standard deviation associated with the constraint on elevation variation from check point to check point; h_b represents the distance between a different set of elevation check points along the actual guideway after or during construction; and σ_b is the standard deviation associated with the accuracy to which the elevation of the profile is being checked or realigned.

Substituting Eq. (5) into Eq. (2) and using the relationships $\bar{\sigma}=\sigma_b/\sigma$ and $\bar{h}=h_b/h$, the temporal spectral density for the guideway irregularities becomes

$$S_v(f)=\frac{4\sigma^2}{(2\pi f)^4}\left(\frac{V}{h}\right)^3\left[\frac{\bar{\sigma}^2}{\bar{h}^3}\,[3-4\cos(2\pi f\bar{h}\frac{h}{V})+\cos(4\pi f\bar{h}\frac{h}{V})]+1-\cos(2\pi f\frac{h}{V})\right] \quad (6)$$

Substituting Eq. (6) into Eq. (4) gives the acceleration spectral density, i.e.,

$$S_a(f)=\frac{4\sigma^2\omega_n^4}{(2\pi f)^4+\omega_n^4}\left(\frac{V}{h}\right)^3\left[\frac{\bar{\sigma}^2}{\bar{h}^3}\,[3-4\cos(2\pi f\bar{h}\frac{h}{V})+\cos(4\pi f\bar{h}\frac{h}{V})]+1-\cos(2\pi f\frac{h}{V})\right] \quad (7)$$

The spectral density for suspension stroke is defined

$$S_r(f)=|G_r(f)|^2 S_v(f) \quad (8)$$

Again from Ref. 1, the transfer function $G_r(f)$ is

$$G_r(f)=\left[\frac{s^2+\sqrt{2}\omega_n s}{s^2+\sqrt{2}\omega_n s+\omega_n^2}\right]_{s=j2\pi f} \quad (9)$$

Substituting Eqs. (9) and (6) into Eq. (8) gives

$$S_r(f)=\frac{1+2(\omega_n/2\pi f)^2}{(2\pi f)^4+\omega_n^4}\,4\sigma^2\left(\frac{V}{h}\right)^3\left[\frac{\bar{\sigma}^2}{\bar{h}^3}\,[3-4\cos(2\pi f\bar{h}\frac{h}{V})+\cos(4\pi f\bar{h}\frac{h}{V})]+1-\cos(2\pi f\frac{h}{V})\right] \quad (10)$$

Using the relationships

$$\langle\Delta\ddot{Y}\rangle^2=\int_0^\infty S_a(f)df \quad (11)$$

and

$$\langle Y_r\rangle^2=\int_0^\infty S_r(f)df \quad (12)$$

the spectral densities were integrated numerically and values of RMS acceleration and RMS suspension stroke were obtained. Figures 2 and 3 show $\Delta\ddot{Y})_{\rm RMS}/\sigma$ and $\Delta Y_r)_{\rm RMS}/\sigma$ plotted as a function of the suspension system natural frequency ω_n and the guideway parameters $\bar{\sigma}$ and $\bar{h}$.

As an example, assume that the vehicle speed is $V=110$ fps (75 mph) and its natural frequency in heave is $\omega_n=1.0$ Hz. Also assume that the original guideway profile was designed on the basis of a distance between points of elevation constraint of $h=44$ ft and that the elevation of the actual constructed profile is to be checked every $h_b=4.4$ ft. Thus, $V/h=2.5$ and $\bar{h}=h_b/h=0.1$. If, in addition, $\bar{\sigma}=\sigma_b/\sigma=1.0$, then from Fig. 2, $\Delta\ddot{Y})_{\rm RMS}/\sigma=0.964$ and $\Delta\ddot{Y}_r)_{\rm RMS}/\sigma=0.988$. Assuming the ride comfort specifications limit $\Delta Y)_{\rm RMS}$ to 0.05 g and that the maximum suspension stroke is 1.0 in., the maximum allowable σ is computed to be 0.622 in. for ride comfort and 1.012 in. for stroke.

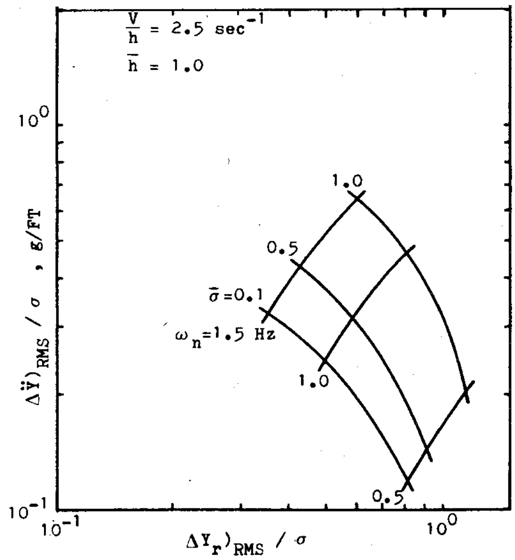

Fig. 3 Effects of guideway design tolerances and constraints on ride comfort and suspension stroke.

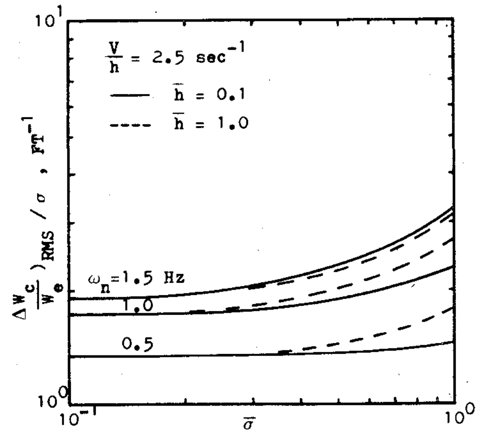

Fig. 4 Impact of guideway design tolerances and constraints on the control flow requirements for an air cushion vehicle.

Realizing that σ represents the standard deviation associated with a random variable, the specific design tolerance designation depends on the type of distribution. If the elevation variations are Gaussian, then the tolerance should be about 3σ. A uniform distribution corresponds to a tolerance of 1.732σ. Thus, for the example given, assuming uniformly distributed elevation variations, the guideway profile must be constrained to $1.732(0.622)=1.077$ in. every 44 ft. Table 1 contains the elevation variation constraint, 1.732σ, for other values of $\bar{h}$ and $\bar{\sigma}$ with $\omega_n=1.0$ Hz and $V/h=2.5$ sec^{-1}.

III. Example Control Requirement Calculations

In order to demonstrate the impact of various guideway design and construction tolerances on suspension control requirements, example calculations for an air cushion suspension are presented in this section of the paper.

The control scheme is to modulate the air flow to the air cushions by means of cushion pressure feedback. Realizing that perturbations in acceleration $\Delta\ddot{Y}$ are proportional to perturbations in cushion pressure, and utilizing specific air cushion design parameters as outlined in Ref. 1, a typical control equation is obtained

$$\Delta W_c(s)/W_e=(27.5/s)(\Delta\ddot{Y}(s)/g) \quad (13)$$

Table 1 Maximum allowable terrain elevation variations (inches) every h ft for $(V/h) = 2.5$ sec^{-1} and $\omega_n = 1.0$ Hz

$\bar{h}$ \ $\bar{\sigma}$	0.1	0.5	1.0
0.1	3.428	1.978	1.077
1.0	3.450	2.931	2.144

Table 2 Ratio of RMS control flow to air cushion equilibrium flow using $\bar{\sigma}$'s shown in Table 1 for $V/h = 2.5$ sec^{-1} and $\omega_n = 1.0$ Hz

$\bar{h}$ \ $\bar{\sigma}$	0.1	0.5	1.0
0.1	0.288	0.180	0.118
1.0	0.290	0.285	0.278

where $\Delta W_c(s)/W_e$ represents the control flow normalized by the total equilibrium flow W_e to the air cushions. As can be determined from Ref. 1, this integral relationship between normalized acceleration and normalized control flow is near optimum and representative for a wide range of air cushion vehicle designs.

The equation for the control flow PSD can be obtained using Eqs. (3), (6), and (13), i.e.,

$$S_w(f) = \left|\frac{27.5}{j2\pi f} G_a(f)\right|^2 S_v(f) = \left(\frac{27.5}{2\pi f}\right)^2 S_a(f) \tag{14}$$

Hence, the normalized RMS control flow is obtained as follows:

$$\left.\frac{\Delta W_c}{W_e}\right)_{\text{RMS}} = \left[\int_0^\infty S_w(f)\,df\right]^{1/2} \tag{15}$$

The integration (Eq. 15) is performed by first substituting Eq. (7) into Eq. (14). Typical values of $(\Delta W_c/W_e)_{\text{RMS}}/\bar{\sigma}$ are presented in Fig. 4 as a function of $\bar{h}$, $\bar{\sigma}$, and ω_n. Specific results relating to the previous example which define the necessary control flow corresponding to the maximum elevation variations in Table 1 are provided in Table 2.

For example, the data in Table 2 for $\bar{\sigma} = 1.0$ and $\bar{h} = 0.1$ indicate that the RMS control flow requirement is 11.8% of the equilibrium flow. If $\bar{\sigma}$ is decreased to 0.5, then the RMS control flow requirement increases to 18.0% of W_e. This increase is the result of relaxing the profile elevation constraint to 1.978 in. every h ft as shown in Table1. For $\bar{\sigma}$ equal to 0.1, the elevation of the profile may deviate up to 3.428 in. every h ft but an RMS control flow of 28.8% of W_e is required to limit the RMS acceleration to 0.05 g.

In conclusion, the data presented indicate the most effective tradeoffs between guideway tolerances and constraints and suspension system requirements to provide acceptable ride comfort while maintaining a low suspension stroke. Since the suspension model is near optimal, the results indicate approximate upper bounds in performance that can be obtained – that is, if the RMS acceleration must be constrained to a particular value and the steady-state power requirements limit the maximum allowable RMS stroke, then the results reported here can be used to determine upper bounds on guideway design tolerances and constraints. As has been demonstrated for an air cushion suspension, the results are beneficial in indicating the necessary control requirements for achieving acceptable performance with a particular vehicle and a particular guideway profile design.

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